DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

120. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics in Boys' High School, Reading, Pa.

How many balls 1 inch in diameter can be put in a cubical box 1 foot in the *clear* each way, putting in the maximum number? [From Greenleaf's *Treatise on Algebra.*]

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa., and H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

The maximum number of balls is not 2149, as given Vol. VII, No. 3, but 2151, as demonstrated below.

Put in 4 rows of 12 balls. Then in the space 8×12 can be put 9 more rows of 12 and 11 alternately; for $8\times\frac{1}{2}\sqrt{3}+1=7.928$.

$8-7.928=.072$ of an inch to spare.

This gives in the first layer 9 rows of 12 each$=108$, and 4 rows of 11 each $=44$. $\therefore$ 152 in all.

In the other space $12\times12\times11$ we can put as before eight layers of 144 each and 7 layers of 121 each.

$\therefore$ Eight layers of $144=1152$
Seven layers of $121=\ \ 847$
One layer of $152\ \ \ =\ \ 152$

Total$=2151$

125. Proposed by F. M. PRIEST, Mona House, St. Louis, Mo.

A Quaker once, we understand
For his three sons laid off his land.
And made three equal circles meet
So as to bound an acre neat.
Now in the center of the acre,
Was found the dwelling of the Quaker;
In centers of the circles round,
A dwelling for each son was found.
Now can you tell by skill or art
How many rods they live apart?

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

The centers of the circles three
With straight lines let united be;
Where touch the arcs, respectively,
These lines will cross the tangency.
Just twice the radius is each line,
And they in trigon space confine
Each circle's sixth and "acre neat,"
No more nor less. With pencil fleet,
From trigon's several vertices
To circles' opposite tangencies,
Respectively, three uprights trace,

And at their intersections place
The Quakers's dwelling. For we find
These uprights are in each combined
Just two-thirds from the trigon-points
And one-third from the tangent-joints.
Each upright we can plainly see
In radius times square root of three; $[r\sqrt{3}]$
And root of three times radius squared $[r^2\sqrt{3}]$
Is trigon's area unimpaired.
A semicircle interjoined
With the Quaker's acre can be coined
An equal to the trigon's space. $[\frac{1}{2}\pi r^2+160]$
Now equal to each other place
The areas of the trigon found;
And if the work is true and sound,
We'll find the half of sixty-three $[31.50+]$
Is a trifle less than in rods should be
The radius of each circled bound
Wherein the sons their dwellings found.
Just twice the radius, or sixty-three, $[63.0+]$
As the rods apart the sons must be.
Two-thirds of the upright, as shown above,
The sons to their father will have to rove;
This distance, in rods, will two decimals run
In one-eighth of two hundred ninety-one. $[36.37+]$
Now we've told by skill and rhyming art
The number of rods they live apart.

II. Solution by J. M. HOWIE, State Normal School, Peru, Neb.; LESLIE J. LOCKE. M. A., Fredonia Institute, Fredonia, Pa.; O. S. WESTCOTT, Chicago, Ill.; J. W. DAPPERT. C. E., Taylorville, Ill.; B. L. REMICK, Bradley Institute, Peoria, Ill.; W. MANZILLA, Langston University, Langston, Okla.

Let ABC be the triangle formed by joining the centers of the farms, CE the altitude. Since the circles are equal, the triangle ACB is equilateral, and therefore $AC=AB=BC=2r$, where r is the radius of the equal circles.

Area of triangle $ABC=\frac{1}{2}AB\times CE=\frac{1}{2}.2r.\sqrt{(4r^2-r^2)}=r^2\sqrt{3}$.

The area of the three circular sectors included in the triangle$=3.\frac{1}{6}\pi r^2=\frac{1}{2}\pi r^2$.

$\therefore$ The area of the curvilinear triangle $EFI=\sqrt{3}r^2-\frac{1}{2}\pi r^2=160$ sq. rods.

$$\therefore r=\sqrt{\left(\frac{320}{2\sqrt{3}-\pi}\right)}=8\sqrt{\left(\frac{5}{2\sqrt{3}-\pi}\right)}=37.7 \text{ rods.}$$

$2r=AC=75.4$ rods=distance between sons' houses, and $\frac{2}{3}\sqrt{3}r=43.5323$ rods=distance from father's to sons' house.

Solved in a similar manner by *G. B. M. ZERR, J. SCHEFFER, C. C. CROSS, H. C. WHITAKER, ELMER SCHUYLER, ALOIS F. KOVARIK, JOHN T. FAIRCHILD, J. M. COLAW, HON. JOSIAH H. DRUMMOND, P. S. BERG, H. I. HOPKINS, COOPER D. SCHMITT*, and *J. O. MAHONEY*.

Solutions of problem 124 were received from *CHAS. C. CROSS, P. S. BERG, J. SCHEFFER, G. B. M. ZERR, ELMER SCHUYLER*, and *H. C. WHITAKER*.

ALGEBRA.

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101. Proposed by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Prove that $(1+2+3+\ldots.+n)+\frac{n}{2!}(1^2+2^2+3^2+\ldots.+n^2)+\frac{n(n-1)}{3!}$